

## Thesis Changes Log

**Name of Candidate:** Oksana Borzenkova

**PhD Program:** Computational and Data Science and Engineering

**Title of Thesis:** Linear optical realization of variational quantum algorithms

**Supervisor:** Prof. Jacob Biamonte

**Co-supervisor:** Dr. Stanislav Straupe

*The thesis document includes the following changes in answer to the external review process.*

*Dear Jury Members, I am grateful for your valuable feedback regarding my work. I appreciate your time and effort in reviewing the thesis material. I have carefully reviewed your comments and recommendations and provided detailed responses to each. Please find them below, together with the corresponding modifications in the final version of the text.*

Reviewer: **Dr. Alexander Korneev**

**Comment:** First of all, I would suggest giving more details concerning the experiments and the processing of the experimental results. E.g. in Chapter 3 I would recommend adding formulas describing how the value of  $E(\theta)$  is calculated from photon counts.

**Answer:** *I have added more experimental details to three chapters. The details of this dimension were in Chapter 4, I moved them to Chapter 3 for the sake of logicity. To facilitate understanding, I have highlighted important terms and questions in the literature review, dividing it into sections for easy navigation.*

**Comment:** In Chapter 5 is a project of a six-photon source with Pockels cells demultiplexer and a source with adjustable delay, but without any information about their performance.

**Answer:** *These schemes have been tested, I added more detailed information about them with graphs, and updated the results table*

**Comment:** There are a certain number of misprints (listed)

**Answer:** *All of them have been corrected now.*

Reviewer: **Dr. Sergey Alyatkin**

**Comment:** // the lack of references in Abstract section is questionable to me. //

**Answer:** *The references added.*

**Comment:** It's not clear why some just introduced abbreviations in the Abstract are deciphered, but most are not?

**Answer:** *This has now been corrected.*

**Comment:** Are the conferences 1. and 2. at page 2 connected to the main topic of the thesis? Maybe it is better to remove them from the list. To me it looks weird to find the conference SNAIA 2018 Paris, December 10-13 with oral talk on "Cortisol Sensor Based on Metal Enhanced Fluorescence".

**Answer:** *This information has been removed.*

**Comment:** At p.5 please change "FWHM — Full width of half-maximum" to "FWHM — Full width at half-maximum" as accepted in the literature.

**Answer:** *This has been corrected.*

**Comment:** Can this strong statement at p.16 be supported by the reference "Any optimization problem can be reformulated into the language of the Hamiltonian (it is not always easy, but possible)..."

**Answer:** *This statement has been corrected and supported by the reference (work by my Supervisor).*

**Comment:** At p.21 "Consequently, the gas of ultracold atoms is utilized as a quantum simulator for solving problems related to the modeling and study of superconductivity, as well as other strongly interacting systems." This sentence leaves the impression that ultracold atoms are strongly interacting. In context of atomic BEC, I thought that ideal Bose gas is a gas of non-interacting or weakly interacting particles.

**Answer:** *Yes, you are right. This has been corrected.*

**Comment:** At p.22,1 don't fully understand this sentence: "The perfect photon source should produce exact states — one photon at a fixed time with high frequency and well-defined properties: spatial, spectral, and temporal" What is meant here by "well-defined spatial properties of one photon" at fixed time? It is possible at all?

**Answer:** *This referred to the characteristics necessary for achieving high indistinguishability of photons, which included the arrival times of the two photons at the beam splitter and the spatial modes, which are significant for the purity of the photons. This has been corrected.*

**Comment:** At p.30 Can the author please provide the explanation to this statement? "The most perspective platform to realize any transformation on qubits is integrated schemes." What are the advantageous of the integrated circuits compared to other platform in terms of states transformations?

**Answer:** *Integrated circuits (ICs) are regarded as a promising platform for implementing transformations on qubits, attributable to their scalability, increased coherence, and precise operational control. The compact architecture of ICs permits the integration of multiple qubits onto a single chip, thereby enhancing connectivity and promoting efficient interactions among qubits. Moreover, ICs capitalize on cost-effective mass production techniques and leverage advancements from classical electronics, rendering them advantageous for the development of practical, reliable, and scalable quantum devices. An explanation has been added to the text, with information concerning integrated circuits segregated into a distinct subsection.*

**Comment:** At p. 53, "The authors showed that increasing the size of a qubit by almost three times reduces the probability of error by just over three percent. However, this does not bother the specialists from Google Quantum AI. They believe that in this way it will be possible to build a fault-tolerant quantum computer, although this will require about a million qubits." Do you mean here the number of qubits instead of the size?

**Answer:** *Yes, you are right. This has been corrected.*

**Comment:** At p.59 in experimental part the HOM visibility drops to 82% compared to 98% in experiment with the fiber splitter. Is a non-ideality of directional coupler (DC7) the only reason of this drop? Is there a room for further optimization with integrated elements?

**Answer:** *It is posited that the imperfections inherent in this particular divider significantly influence the overall performance. It is essential to acknowledge, however, that imperfections present in other directional couplers (DCs) within the circuit likewise affect photon indistinguishability. For instance, dividers DC1-5 might not efficiently channel photons into modes 3 and 4, while measurements obtained with dividers DC9-13 may also exhibit imperfections. Crucially, in the context of quantum mitigation, the precise origins of these errors are less pertinent than their cumulative effect on observed outcomes. Consequently, a comprehensive investigation into the individual sources of error is deemed unnecessary at this juncture ( Section 4.6 discusses expressions that are independent of the specific modes being analyzed, whether these pertain to polarization, spatial, wavelength, or other parameters). Future plans entail further experiments with the chips, and the quality of chip manufacturing in our laboratory has notably improved throughout the progression of this thesis.*

Reviewer: **Vladimir V. Palyulin**

**Comment:** From the Fig 4.8 it seems that the relative error decreases monotonously with the delay of the error mitigation. The question is whether there is a lower limit for this method of error reduction? Because currently it looks that the best strategy is to delay the mitigation as much as possible which is rather counter-intuitive.

**Answer:** *In this context, it is imperative to acknowledge that the number of measurements, which dictates the duration of the experiment, limits us rather than the iterations themselves. Consequently, if we possess 120 measurements, the algorithm is constrained to only 10 iterations if mitigation commences at the outset (calculated as 3 measurements per basis \* 2 for Simultaneous Perturbation Stochastic Approximation (SPSA) \* 2 for mitigation). Conversely, by postponing mitigation to the final two steps, we can achieve 16 iterations without mitigation, followed by 2 iterations with it, amounting to 18 total iterations. A higher number of unmitigated iterations enhances the accuracy of the algorithm's convergence. This is due to the observation that during the initial iterations, noise influences the algorithm's convergence as significantly as the initial conditions do, rendering mitigation ineffective at this juncture. However, in the later iterations, when noise exerts a considerable impact, mitigation proves to be beneficial.*

**Comment:** Apart from the question I have spotted some minor typos such as initials of SA Akhmanov in Ref. 4 (should be SA not CA) or “unimoal” on page 47, the conflicting notations (H is used both for a horizontal polarisation and a Hamiltonian, V is both a vertical polarisation and a HOM visibility). Also, some figures such as, for instance, 4.4 and 4.5 do require an improvement of captions. The latter have to be more self explanatory.

**Answer:** *Regrettably, distinguishing the notation of the Hamiltonian, visibility, and horizontally and vertically polarizations presents challenges, given their widespread acceptance in academic discourse. However, efforts have been made to distinguish these notation within the text to mitigate potential misinterpretations. All other inaccuracies have been rectified.*

Reviewer: **Dr. Konstantin Katamadze**

**Comment:** Do variational quantum algorithms provide a quantum advantage? What scale of the quantum computation system will be enough for it?

**Answer:** *Variational Quantum Algorithms (VQAs) demonstrate significant potential for quantum advantages in addressing complex problems, particularly as quantum technology continues to advance. Despite facing challenges in scalability and practical constraints such as temporal and efficiency limitations, VQAs are anticipated to outperform classical algorithms. The optimization of these algorithms through enhanced sampling methods, parameter reduction, and efficient training of*

*quantum neural networks engenders optimism regarding the exploitation of the unique capabilities inherent in quantum computing. Moreover, the integration of quantum methodologies with machine learning via a hybrid quantum-classical paradigm can result in pioneering advancements across diverse fields. I added the paragraph with the examples and references (page 21)*

**Comment:** All the presented implementations of quantum algorithms on the linear optical platform are based on principally non-scalable two-qubit gates. How can the obtained results be applied to scalable architectures?

**Answer:** *Variational quantum algorithms (VQAs) face scalability challenges due to increasing costs of gradient evaluation with the escalation in the number of parameters, resulting in extended runtimes and difficulties in identifying practical solutions. The expansion of qubits and parameters adds further pressure on the duration of gate operations and efforts for error mitigation. In response, researchers advocate for the adoption of more efficient sampling methodologies for gradient estimation, ansatz designs that conserve parameters, and hybrid quantum-classical approaches with supplementary classical processing. Additionally, advancing error-correcting codes and enhancing qubit quality to prolong coherence times may facilitate VQAs in addressing more intricate problems. I added a subsection about the limitations of VQAs and ways to overcome them (page 25).*

**Comment:** In Chapter 4, photon distinguishability was considered as the main source of errors since it decreases the visibility of HOM interference (82% using the optical circuit). At the same time, it was mentioned that the HOM interference in a fiber beam splitter was 98%, which means the main source of errors is not photon distinguishability but the optical circuit's imperfection. Is it correct to mitigate one imperfection by considering the variation of another one? Why was photon distinguishability varied by polarization, and not by the time delay? Can this polarization change affect the phases and splitting ratios of the optical circuit?

**Answer:** *Section 4.6 discusses expressions that are independent of the specific modes being analyzed, whether these pertain to polarization, spatial, wavelength, or other parameters. This independence results in a nearly linear dependence on visibility, which is applicable to our circumstances. The experimental calibration of chips indicates that variations in polarization within the experimental range, specifically a 10-degree rotation of the HWP, do not significantly impact the division factors.*

**Comment:** How does eq (4.10) for unitary matrix fidelity reflect the real quality of the optical circuit, since it does not include the phases of the unitary matrix elements? Does the high value of this fidelity (99.18%) mean that not only the splitting ratios but also the passive phases of the produced circuit fit well with the desired one? In the conclusion, it was mentioned that the fidelity of the two-qubit entangling CNOT gate is 94.4%. How was this value obtained? How does it fit with the 82% HOM interference visibility?

**Answer:** *Thank you for this valuable observation. We have demonstrated our capability to produce high-quality chips and to execute two-qubit operations with high fidelity. However, in our rally experiment, we utilized a previous iteration of the same chip, which proved to be less successful. Nevertheless, by employing quantum error mitigation techniques, we were able to accurately achieve a zeroth-order result. This information has been incorporated into the text on page 89, section 4.5.*

**Comment:** The multi-photon source, presented in Fig. 5.2, has 8 outputs. The caption of the figure is "Experimental setup with six photons". In the table presented in Fig. 5.3, no more than 4-photon coincidence is presented. So, how many photons can this source produce? Does it use different crystals in different parts? Why?

**Answer:** *Chapter 5 provides a comprehensive account of the experimental procedure involved in the assembly and configuration of the sources. Table 5.1 is presented in a detailed and informative manner on page 95. Initially, the experimental framework assumes the configuration of eight*

channels; however, in our experiment, six channels were configured. The experimental outcomes aimed to assess events involving four and six photons. The crystals employed in our experiments were sourced from existing laboratory inventories, as they were not custom-manufactured to ensure optimal parameter uniformity, a process which typically requires over a year for production and delivery.

Reviewer: **Yong-Su Kim**

**Comment:** This thesis reports experimental implementation of photonic variational quantum eigensolver and related analysis. From the contents of the thesis, I can see that the author seems to have deep knowledge on both photonic quantum information and variational quantum algorithms. However, I also found there are many parts that need to be revised before final defense. I am attaching my detailed comments and these should be addressed in the final version of thesis

**Answer:** Upon reviewing the notice, I isolated Chapter 1 to address the variation of quantum algorithms and adiabatic evolution separately. In the same chapter, I introduced a section concerning the limitations associated with the variation of quantum algorithms. The portion describing various platforms was revised during the review process, incorporating recommendations for narrative structuring and integrating suggested links. The description of single-photon sources was divided into two sections, with a comparison provided on page 39. A segment from Section 2.4, which discusses the application of quantum algorithms, was moved to the introduction, and Section 2.3.4 was accordingly adjusted. On page 61, attention was given to distinguishing between QEC and QEM, with a note that association with QEM should include purification method. All minor textual amendments were executed, and Chapter 5 was enriched with more comprehensive experimental data. Two questions remain pending for response:

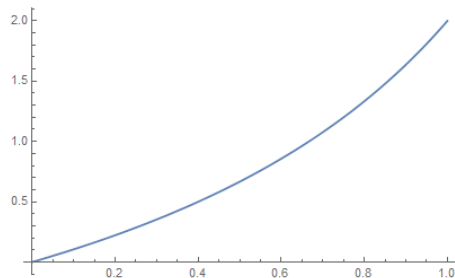
**Figure 3.6 (page 46, now page 76). In your simulation why you can get values smaller than theoretical minimum?**

These are statistical fluctuations arising from the finite number of measurements. In the processing, it was considered that the analysis is based on limited statistical data, thus resulting in an estimation of the medium energy that varies around the true value.

**Section 4.9 page 67 (now section 4.6 page 89) noise level  $\epsilon$  is not too small in experiment. And (next page) what happens for large  $\epsilon$ ?**

Figure 4.6, previously referred to as Figure 4.7, illustrates the phenomena observed at elevated noise levels. Formula 4.25, formerly Formula 4.24, provides a detailed description of this dependence. Upon constructing a dependence schedule for this formula, it becomes evident that for values of  $\epsilon$  less than 0.4, the relationship approximates linearity.

Plot  $\left[\frac{2\epsilon}{2-\epsilon}, \{\epsilon, 0, 1\}\right]$



Reviewer: **Prof. Ivan Oseledets**

**Comment:** The thesis could benefit from a more in-depth discussion of the limitations and challenges encountered in the experimental work, as well as potential avenues for future improvements and scalability of the photonic quantum processor.

**Answer:** *I have incorporated Section 2.1.4, which elucidates the limitations in the variation of quantum algorithms employed in this study, as well as the methodologies designed to surmount these constraints. These limitations, including phenomena such as barren plateaus, are pertinent to the experimental component of the research.*

*Furthermore, the scalability of photon-based computational systems is not thoroughly examined within this work. Nonetheless, Chapter 5 delineates strategies for the scaling of single-photon sources and the enhancement of their properties, aligning with my primary experimental focus.*

**Comment:** While the thesis demonstrates the effectiveness of the ZNE method, a more comprehensive analysis of the trade-offs between the improved accuracy and the increased experiment duration or estimator variance would provide a more complete picture of the method's practical applicability.

**Answer:** *The investigation into the balance between the precision of the mean value and the variability within the scope of this work is not addressed in-depth. In instances of linear extrapolation, this relationship is defined by Formula 4.4. Additionally, for scenarios involving elevated noise levels, simulations are conducted to demonstrate the dispersion for adjusted energy values. Nonetheless, this topic merits further exploration in subsequent research.*

**Comment:** ZNE method focuses on linear extrapolation; in numerical analysis, this reminds me of so called 'Richardson extrapolation' which approximates the solution of a partial differential equation (PDE) given on different grids, but the assumption is more general:  $f(h) = f_0 + O(h^c)$ . Are there any models that have non-linear dependence on the noise level?

**Answer:** *In our experiment, noise exhibits linear behavior only at specific noise levels (Formula 4.25, formerly Formula 4.24). The graph (Figure 4.6, previously referred to as Figure 4.7) illustrates that for noise levels exceeding 0.4, the linear approximation becomes less effective. Furthermore, in certain quantum simulations or complex quantum systems, noise may induce state-dependent errors that do not scale linearly with the noise level. Models employed in quantum state tomography or quantum process tomography often incorporate non-linearities in error modeling, contingent upon the characteristics of the quantum operations involved.*

*Beyond linear approximations, these models frequently utilize exponential approaches or select models adaptive to the specific task at hand, as detailed in studies such as "Digital Zero Noise Extrapolation for Quantum Mitigation" by Tudor Gurgica-Tiron, et al.*

**Comment:** It would be interesting to highlight personal contribution of the author, for example, for the experimental setup.

**Answer:** *All single-photon sources discussed in the article were assembled and configured by me, and I also conducted measurements of their characteristics. Furthermore, I was responsible for the complete assembly and initiation of the systems, including the calibration processes. My colleague, Ilya Kondratyev, assisted in the calibration and alignment of the chips, while Gleb Struchalin provided support with simulations and control codes. The integrated chips were manufactured with the assistance of Nikolay Skryabin.*