

Jury Member Report – Doctor of Philosophy thesis.

Name of Candidate: Andrii Liashyk

PhD Program: Mathematics and Mechanics

Title of Thesis: Bethe vectors and their scalar products in quantum integrable models

Supervisor: Professor Anton Zabrodin

Date of Thesis Defense: 20 January 2020

Name of the Reviewer: Ivan Kostov

I confirm the absence of any conflict of interest	<p>Signature:</p>  <p>21-12-2019 Date: DD-MM-YYYY</p>
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The purpose of this report is to obtain an independent review from the members of PhD defense Jury before the thesis defense. The members of PhD defense Jury are asked to submit signed copy of the report at least 30 days prior the thesis defense. The Reviewers are asked to bring a copy of the completed report to the thesis defense and to discuss the contents of each report with each other before the thesis defense.

If the reviewers have any queries about the thesis which they wish to raise in advance, please contact the Chair of the Jury.

Reviewer's Report

Reviewers report should contain the following items:

- Brief evaluation of the thesis quality and overall structure of the dissertation.
- The relevance of the topic of dissertation work to its actual content
- The relevance of the methods used in the dissertation
- The scientific significance of the results obtained and their compliance with the international level and current state of the art
- The relevance of the obtained results to applications (if applicable)
- The quality of publications

The summary of issues to be addressed before/during the thesis defense

- Small linguistic issues (already communicated to the Author) to be fixed.
- The part of the Introduction concerning the Yangian Double is too sketchy. It is necessary at least to explain what specifies the two components of the Yangian double, T^+ and T^- , and how they are related.
- The bibliography should be completed

Provisional Recommendation

I recommend that the candidate should defend the thesis by means of a formal thesis defense

I recommend that the candidate should defend the thesis by means of a formal thesis defense only after appropriate changes would be introduced in candidate's thesis according to the recommendations of the present report

The thesis is not acceptable and I recommend that the candidate be exempt from the formal thesis defense

Paris, 26 December 2019

A handwritten signature in blue ink, appearing to read "T. Kostov", is written on a light grey rectangular background.



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December 26, 2019

Referee report to the PhD thesis
“BETHE VECTORS AND THEIR SCALAR PRODUCTS
IN QUANTUM INTEGRABLE MODELS ”
by
Andrii LIASHYK

The PhD thesis “Bethe vectors and their scalar products in quantum integrable models” by Andrii Liashyk addresses aspects of the Algebraic Bethe Ansatz for quantum integrable models with higher rank (super) symmetry, relevant for computing form factors and correlation functions. The Thesis is based on five scientific publications co-authored by the candidate and is organized in six chapters, five of which are reproductions of the above-mentioned scientific articles. The Thesis terminates by Conclusions.

The first chapter plays the rôle of introduction and gives a brief description of the problems, the methods and the results reported in the next five chapters. Although this chapter covers all the essential notions and techniques used in this Thesis and demonstrates comprehensive understanding of the subject, its minimalist style renders it accessible only for readers already familiar with the mathematical aspects of the quantum inverse scattering method. In particular, the part concerning the Yangian double is too sketchy. It is necessary at least to explain what specifies the two components of the Yangian double, T^\pm , and how they are related. There are also some stylistic imperfections to be addressed. The bibliography of 27 references is far from being complete.

The next five chapters reproduce the scientific papers the Thesis is based on. Before each chapter the candidate specifies his original contribution.

The second chapter is devoted to the study of the Bethe vectors in integrable models with $gl(m|n)$ Yangian symmetry. The off-shell Bethe vectors are constructed by generalizing the method of projections, advanced by Khoroshkin and Pakuliak, which uses Drinfeld current realization of the Yangian. The Bethe vectors are constructed in two different but equivalent ways as products of total currents applied to the pseudo-vacuum, with subsequent projection to the positive part. The technically challenging part is the projection, which is more subtle

in case of supersymmetry. The projection is computed using a pair of co-product properties of the Bethe vectors. This gives explicit expressions of the Bethe vectors in terms of the elements of the monodromy matrix acting on a singular vector. The original contribution of the candidate consists in computing the action of the monodromy matrix on thus constructed Bethe vectors and demonstrating that the Bethe vectors become eigenvectors of the transfer matrix if they are on shell, i.e. if the Bethe parameters satisfy the Bethe-Yang equations. The results of this section give the basis for the further developments described in the subsequent chapters.

The third chapter concerns the problem of computing scalar products of off-shell Bethe vectors in integrable models with $gl(m|n)$ supersymmetry. The main result is a formula for the scalar product of generic Bethe vectors in terms of a sum over partitions of Bethe parameters, generalizing the Korepin sum formula. The derivation is based on a co-product formula for the Bethe vectors. Recursion formulae are derived for the highest coefficient in the sum, the higher-rank analogue of the Izergin determinant. The original contribution of the candidate is the proof of the recurrence relations for the dual Bethe vectors and an essential part of the proof of the sum formula.

The fourth chapter discusses the evaluation of the norm of on-shell Bethe vectors in generalized integrable models with $gl(m|n)$ -invariant R-matrix. The principal result is the proof of the Gaudin hypothesis that the square of the norm is equal to the Jacobian for the map from the scattering phases to the Bethe parameters. The derivation is based on the sum formula for the scalar product of generic Bethe vectors derived in Chapter 3. The norm squared is obtained by taking the limit of coinciding Bethe parameters. The limit is performed by applying a formula, proved by the candidate, which expresses the scalar product with a pair of Bethe parameters identified in terms of a scalar product in a generalized model with modified functional parameters.

In the fifth chapter, the results obtained in the previous chapters (recursion formulas for generic Bethe vectors, sum formula for their scalar products, recursion formula for the highest coefficient) are formulated for the generalized model whose symmetry is a q -deformation of the $gl(n)$ algebra. When the Bethe vectors are on-shell, it is shown that their norm takes the form of a Gaudin determinant. The original contribution of the candidate concerns the recurrence relations for the dual Bethe vectors and the proof of the Gaudin conjecture for the norm.

The sixth chapter investigates the symmetries of $gl(N)$ -invariant off-shell Bethe vectors generated by the automorphism of the RTT algebra obtained by replacing the monodromy matrix with its inverse. It is shown that the Bethe vector is invariant under certain reordering and shifts of the Bethe parameters. This highly non-trivial symmetry is proven by using the Drinfeld current realization and the projection method. The contribution of the candidate consists in proving the main theorem as well as a symmetry property for the highest coefficient in the sum formula for the scalar product.

As a manuscript this Thesis would definitely benefit by a more detailed presentation of the current state of art and a more general introduction. Furthermore the five articles contain inevitable repetitions which would have been avoided if they were replaced by a unifying text. The limited time excludes substantial changes, but some minimal improvements according to my comments concerning the Introduction could be implemented before the date of the defence. Of course these remarks should by no means diminish the quality of the research

presented in this Thesis, which is the most important criterium for obtaining the academic degree.

Concerning the quality of the research, the results obtained by the candidate are original and represent valuable addition to the analytic methods related to the Algebraic Bethe Ansatz. The sum formulas will certainly find application in the computation of the correlation functions in integrable models with higher rank symmetry or with supersymmetry such as the t-J model and the spin-chain for the maximally supersymmetric Yang-Mills theory. The Drinfeld current approach, which is the backbone of this Thesis, represents not only a powerful computational tool, but also a unifying conceptual scheme which can be adapted to a variety of integrable structures with higher rank symmetry. This is partially done in subsequent papers co-authored by the candidate, which were not included in this Thesis.

In conclusion, to my opinion the research presented in this Thesis is of very high quality by international standards and I recommend that Mr Andrii Liashyk is granted the academic degree of Doctor of Philosophy (PhD).

Sincerely,



Ivan Kostov