Skoltech
Skolkovo Institute of Science and Technology

Jury Member Report – Doctor of Philosophy thesis.

Name of Candidate: Polovnikov Kirill

PhD Program: Physics

Title of Thesis: On connection between sparse graphs and hyperbolic geometry

Supervisor: Professor Mikhail Gelfand, Skoltech

Professor Sergey Nechaev, Interdisciplinary Scientific Center Poncelet

Name of the Reviewer: Professor Vladik Avetisov, N.N. Semenov Federal Research Center of Chemical Physics of the Russian Academy of Sciences

I confirm the absence of any conflict of interest	**Signature:**  **12-08-2020**

The purpose of this report is to obtain an independent review from the members of PhD defense Jury before the thesis defense. The members of PhD defense Jury are asked to submit signed copy of the report at least 30 days prior the thesis defense. The Reviewers are asked to bring a copy of the completed report to the thesis defense and to discuss the contents of each report with each other before the thesis defense.

If the reviewers have any queries about the thesis which they wish to raise in advance, please contact the Chair of the Jury.

Reviewer's Report

Reviewers report should contain the following items:

- Brief evaluation of the thesis quality and overall structure of the dissertation.
- The relevance of the topic of dissertation work to its actual content
- The relevance of the methods used in the dissertation
- The scientific significance of the results obtained and their compliance with the international level and current state of the art
- The relevance of the obtained results to applications (if applicable)
- The quality of publications

The summary of issues to be addressed before/during the thesis defense

The Skoltech Doctoral Thesis by Kirill Polovnikov is represented by his six articles published together with co-authors over the past three years. These publications form the bulk of the 184-page dissertation. Preprints of these articles are available in six chapters, from Chapter 2 to Chapter 7, each article in a separate Chapter. Each of these chapters contains extended annotations reflecting the topic and the issue under the study, the idea of how to solve the problem, the results, and their novelty in the field of physics. Since the articles have been written with the co-authors, the Candidate indicates his personal contribution to each article. As a witness to the formation of these issues for these years, I confirm that the personal contribution of the Candidate is accurately reflected.

Chapter 1 contains a comprehensive introduction to the dissertation work and precedes the preprints. Chapter 8 contains a short summary and completes the dissertation.

In the Introduction, the Candidate gives an insight into what these six articles have in common. From his point of view, these publications, in fact, are related to the statistics of rare events and the relationship between this statistics and hyperbolic geometry. Half of the articles are devoted to the spectral properties of sparse random graphs. It is obvious that sparse random graphs have a lot to do with statistics of rare events. The rest of the articles devoted to random walks under topological constraints, embedding of an exponentially growing surface into 3-dimensional (Euclidean) space, and time series of the cryptocurrency stock market have a rather covert relation to the statistics of rare events and does not look as straightforward as for sparse graphs.

In the introduction, the Candidate expounds arguments in support of his position. As far as I understand, he is promoting the idea that discreteness and scale invariance are inherent in the statistics of rare events. Therefore, the probabilistic characteristics of rare events ensembles should be described by functions with the same characteristics. Generally speaking, this idea has sounded plausible among the Candidate's co-authors; however, as far as I know, concrete incarnations of this idea, such as the statement that the average spectral density of the ensemble of sparse graphs can be approximated by the functions, which are modular forms with respect to the modular group $SL(2,\mathcal{Z})$, appeared with the Candidate's direct incorporation into these studies.

The Candidate's logic and argumentations demonstrates deep knowledge of common theoretical tools of statistical physics and probability and spectral graph theories in addition to the knowledge of specialized mathematical topics related to hyperbolic and ultrametric spaces. The fact that the author's logic and argumentation are woven into the fabric of commentary to each publication looks especially attractive.

In my opinion, the high scientific level of Kirill Polovnikov's publications and the high quality of the article presentation in the dissertation work is beyond doubt. Non-trivial relationships between the articles are logically stated and well-argued in the dissertation work. The style and manner of the author's explanations undoubtedly reflect his high professional qualifications and deep understanding of the discussed issues.

According to the formal rules, I should give an assessment of the relevance of the topic of the dissertation to its actual content. For the dissertation work by Kirill Polovnikov, this assessment is easy to make as it is enough to compare the content of the presented articles and the title of the dissertation. I will discuss the articles as they are listed in the bibliography on page 5 of the dissertation:

In article [1], the number-theoretic properties of the spectral density of exponentially weighted ensemble of linear polymer chains are discussed. It is shown that the eigenvalue statistics of adjacency matrices in the sparse regime can be described by the popcorn (Thomae) function, discontinuous in the dense set of rational numbers. The authors suggest a continuous approximation of the popcorn function and demonstrate that the ultrametric structure of the popcorn-like distributions is ultimately connected with hidden SL(2,Z) modular symmetry. Thus, in this article we have an eloquent example demonstrating the connection between the discreetness of the statistics of rare events and the hyperbolic geometry of the phase space of the system.

In article [2], the authors demonstrate the fact that the boundary profile of a tissue growing by means of exponential division of its peripheral cells is described by the two-dimensional eikonal equation, which provides the geometric optic approximation for the wave front propagating in a medium with an inhomogeneous refraction coefficient. They demonstrate that the elastic energy of the buckled tissue can be expressed through the Dedekind η-function. This example also indicates that the hyperbolic space of the system states naturally arises in the cases when the system realizes a vanishingly small part of an occupied space.

Article [3] discusses rare event statistics related to fluctuations of a two-dimensional random walk stretched above semicircle or triangle obstacles. As in articles [1] and [2], the rare realizations caused by geometric constraints are discussed and it is shown how rare statistics yields the scale-invariance characteristics of observables.

Article [4] is another example of how topological constraints cause fractal properties of the observables. A modular network model of the cryptocurrency market is proposed and discussed in this article. This example may seem somewhat exotic, but it allows the authors to use the long-time evolution of the real observables for numerical estimates supporting an underlying scale-invariance hypothesis.

Finally, the last two papers [5] and [6] discuss the block-hierarchical presentation of the genome contact map. Such a presentation is naturally associated with tree-like branching graphs, scale-free modularity, rare realizations, and hyperbolic spaces.

Thus, despite the fact that the six articles selected by the Candidate for his dissertation work consider various objects, they are related to each other and the topic of the dissertation actually reflects the content of the publications.

The relevance of the methods used in the dissertation, and the significance of the results obtained, in addition to their compliance with the international level, cannot be questioned. The dissertation work presents theoretical researches of high scientific level. The main results

and defended statements are published in high-ranking international journals. This fact, in my opinion, is enough to justify the findings of the thesis.

About the relevance of the obtained results to applications. At least four publications suggest particular applications. Article [3] examines the diffusion of nanosized macromolecules through narrow pores. This topic has variety modern applications, such as in cellular biophysics and nanotechnology. Article [4] directly addresses to the cryptocurrency market. Articles [5] and [6] are focused on a hot question on the DNA package in cell nuclei: the answer is eagerly awaited in genetics and biomedicine.

In general, the dissertation work, "On the connection between sparse graphs and hyperbolic geometry", by Kirill Polovnikov is a high-quality PhD thesis. Based on a set of interrelated researches carried out by the Candidate, he developed well-argued theoretical provisions, which can be assessed as a scientific achievement.

I recommend that the Candidate should defend his thesis by means of a formal thesis defense.

Provisional Recommendation

X ***I recommend that the candidate should defend the thesis by means of a formal thesis defense***

☐ *I recommend that the candidate should defend the thesis by means of a formal thesis defense only after appropriate changes would be introduced in candidate's thesis according to the recommendations of the present report*

☐ *The thesis is not acceptable and I recommend that the candidate be exempt from the formal thesis defense*