Skoltech
Skolkovo Institute of Science and Technology

Jury Member Report – Doctor of Philosophy thesis.

Name of Candidate: Polovnikov Kirill

PhD Program: Physics

Title of Thesis: On connection between sparse graphs and hyperbolic geometry

Supervisor: Professor Mikhail Gelfand, Skoltech

Professor Sergey Nechaev, Interdisciplinary Scientific Center Poncelet

Name of the Reviewer: Professor Nikolay Brilliantov

I confirm the absence of any conflict of interest (Alternatively, Reviewer can formulate a possible conflict)	**Signature:**  **Date: 14/08/2020**

The purpose of this report is to obtain an independent review from the members of PhD defense Jury before the thesis defense. The members of PhD defense Jury are asked to submit signed copy of the report at least 30 days prior the thesis defense. The Reviewers are asked to bring a copy of the completed report to the thesis defense and to discuss the contents of each report with each other before the thesis defense.

If the reviewers have any queries about the thesis which they wish to raise in advance, please contact the Chair of the Jury.

Reviewer's Report

Reviewers report should contain the following items:

- Brief evaluation of the thesis quality and overall structure of the dissertation.
- The relevance of the topic of dissertation work to its actual content
- The relevance of the methods used in the dissertation
- The scientific significance of the results obtained and their compliance with the international level and current state of the art
- The relevance of the obtained results to applications (if applicable)
- The quality of publications

The summary of issues to be addressed before/during the thesis defense

The Thesis of Kirill Polovnikov entitled "On connection between sparse graphs and hyperbolic geometry" is dedicated to the spectral properties of sparse random networks and their application to the community detection. The dissertation is based on six research papers. Three papers are mainly addressing theoretical issues, while the other three ones focus on the spectral analyses of real networks. The structure of the dissertation consists of an abstract, table of contents, introduction, chapters reflecting the content of the published papers and conclusion. The overall quality of the papers is rather high, which is confirmed by the ranking of the journals – five papers are published in Q1 journals.

In the First Chapter I the author provides a detailed introduction to the subject of the thesis. Here he also explains the main line of the research which unites seemingly different topics of the thesis. The level of the detail is adequate as it helps to understand the overall approach and introduce the notations and major ideas, which are developed in the subsequent chapters.

In Second Chapter Kirill addresses the fundamental connection between the averaged spectral density of an ensemble of sparse Erdos-Renyi graphs and action of a special modular group in the hyperbolic space. It is shown how the spectral densities of linear and regular Bethe tree subgraphs can be approximated by the Dedekind η-function close to the real axis. At the same time, the native ultrametricity of the spectral density persists when one considers the full ensemble of random Erdos-Renyi subgraphs, including irregular trees. The tails of the spectral density are shown to demonstrate the Lifshitz singularities, corresponding to localization in 1D. Interestingly, the asymptotic behavior at all inner rational points of the support corresponds to the Lifshitz singularity in 2D, which is attributed to the general properties of the eigenvalues of Wigner matrices repelling as a two-dimensional Coulomb gas confined on a line. It is illustrated in this Chapter that very similar mathematical objects underlie very different physical phenomena which make the fundamental research very practical.

In the Third Chapter, the manifestation of the hyperbolic geometry in sparse graphs is discussed in the context of isometric embedding of an exponentially growing Bethe tree into the hyperbolic two-dimensional space. The conformal transformation between the Euclidean and hyperbolic fundamental domains allows to study the shapes of planar surfaces, where the instabilities are evoked by the incompatibility of the local growth protocol with the ambient metric. These geometric arguments agree well with numerous of energetic approaches to buckling of thin membranes, where the stiffness is controlled by the effective bending rigidity. Essentially, the geometric approach uncovers the fundamental geometrical constraints, provided by the Hilbert's embedding theorem. The shapes of the growing objects are shown to be described by the eikonal-type equation with the refraction index, which is expressed through the Dedekind η-function. Again, the applicability of basic theoretical techniques to very different processes, like light propagation and plants growth is demonstrated.

The emergence of Lifshitz tails in spectra of various systems with disorder is deeply connected with the KPZ universality class. As an example of such systems, the stretched Brownian paths emanating boundaries of different shapes are considered in the Fourth Chapter. The fluctuations of these paths exhibit the 1D KPZ exponent, when the boundary is circular, contrary to the

Gaussian exponent typical for the unconstrained Brownian paths. Interestingly, in this regime the asymptotic of the PDF of fluctuation is shown to be described by the tail of the Tracy-Widom distribution, strongly suggesting that the resemblance with the fluctuations of the eigenvalues of the ensemble of random matrices.

In the rest of the thesis practical applications of the spectral graph theory in the sparse regime, elaborated in the first three chapters are presented. Namely, in the Fifth Chapter the author analyzes the core-periphery segregation of the cryptocurrency market and the topologically-associated domains in the Sixth Chapter. In the Seventh Chapter of the thesis the topologically-associated compartments in single cell Hi-C networks are addressed. To the aim of chromatin communities, Kirill has developed two algorithms, which identify communities in sparse Erdos-Renyi graphs using non-backtracking random walks. Impressively, it has allowed to infer compartments in chromatin folding at the single cell level, which has never been done before; this result is especially important for the biological applications. The boundaries of the established domains are shown to be filled with epigenetic markers, which prominently demonstrates a biological relevance of the communities. These results are published in collaboration with biologists in two prestigious journals, which belong to the Nature group.

The overall quality of the Thesis is rather high. Moreover, a couple of new, interesting and scientifically important results have been obtained. Noteworthy, the subject of the thesis (and of the respective papers) is completely different from the PhD Thesis previously defended by Kirill in Moscow State University. The present Thesis reflects an independent research of the applicant with Profs. Sergei Nechaev and Mikhail Gelfand.

The author demonstrates a variety of skills of a theoretical physicist together with a deep understanding of an addressed phenomenon. The intention to apply the most modern theoretical methods to analyse practical problems, including important biological problems, proves to be a very productive. I admire the elegance of some mathematical approaches presented in the Thesis.

Although I consider the Thesis as a solid scientific work satisfying all the criteria of a PhD qualification document, I still have comments listed below.

1) While the author has made some efforts to put all the studies under one umbrella in the Introduction, I expect the same to be done in the Conclusion, where the obtained results are to be discussed from a unified point of view. This unfortunately has not been done -- the author just repeats the main finding of the Thesis. I admit that it is not a simple task, but the respective attempt would be very beneficial.
2) (i) Can one apply the path integral approach to the problem of random motion around the forbidden semicircular (or triangular) area? I expect that this technique with an according fictitious potential that mimics the presence of the semicircle (or triangle) would be more efficient than the respective PDE used in the Thesis.
   (ii) Another question which refers to this model: Will the addition of the third dimension change the result? I would expect that for the case of a cylinder (with an axis normal to the studied plane) the results will persist, while for a semi-sphere a new scaling will appear, due to the multiplicity of equivalent trajectories. Can the author comment on this?

3) I do not think that it was a good idea to put a complete text of the article submitted to Nature Communications, with numerous biological data (which has nothing to do with the main topic of the Thesis). Instead, one should write in the according Chapter of the Thesis, a much more extended discussion of the results that refer to the mathematical part of the work. Moreover, a brief explanation of some biological terms and concepts would be very helpful. For instance, the author writes "with 10 kb resolution" or "at a scale of 100 kb…". It is not clear for a non-specialist, what is the meaning of such statements. That is, whether the resolution is sufficient, or whether the scale is large or small as compared to the complete genome?
4) The statement "an important contribution of stochastic processes to the formation of the Drosophila 3D genome" is very vague. Why it is important? Is it measurable? I believe that it is better to avoid such statements in scientific texts. Indeed, it is very difficult to discriminate between stochastic and deterministic process – the nonlinear dynamics provides numerous examples of chaotic trajectories, generated by deterministic dynamics.
5) The usage of different fonts and formats of the supplemented articles in different parts of the text is irritating. Why not to use a unified font and format?

These comments refer however to the exposition of the material and do not undermine the overall high quality of the Thesis, which satisfies all the requirements of Skoltech for the PhD defenses. Based on this I strongly recommend the Committee to award Kirill Polovnikov the Skoltech PhD degree in Theoretical Physics.

Faithfully Yours

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Provisional Recommendation

X☐ *I recommend that the candidate should defend the thesis by means of a formal thesis defense*
☐ *I recommend that the candidate should defend the thesis by means of a formal thesis defense only after appropriate changes would be introduced in candidate's thesis according to the recommendations of the present report*
☐ *The thesis is not acceptable and I recommend that the candidate be exempt from the formal thesis defense*