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Report on the PhD thesis "Twisted representations of toroidal algebras and their applications" by Roman Gonin

Infinite dimensional Lie algebras and their quantum analogues play a very important role in modern mathematics and mathematical physics due to numerous applications in quantum field theory, algebraic geometry, combinatorics and integrable systems. The importance stems from the various constructions of actions of the infinite dimensional algebras on the objects scientists are interested in: spaces of states of a quantum theory, spaces of solutions of differential or difference equations, algebraic varieties, moduli spaces, etc. It is this very desirable to study the representation theory of infinite-dimensional algebras.

The thesis of Roman Gonin deals with the representation theory of quantum toroidal algebra gl_1. This algebra and its representations proved to be important in the theory of Macdonald polynomials (via the Double Affine Hecke Algebra formalism), in conformal field theory (via bosonizations, vertex operators and the deformed W -algebras), in the theory of the moduli spaces of the torsion free sheaves on the projective plane (via the Gorsky-Negut conjecture). The higher rank analogues of the quantum toroidal algebra gl_1 have been studied for more than 20 years, however, the gl_1 case has not been considered until recently. The Roman Gonin thesis is an important contribution to the field. The main results of the thesis are explicit realizations of twisted Fock modules of the quantum toroidal algebra, the construction of the action of the Virasoro algebra on the level one representations of quantum affine sl(2) and the semi-infinite construction of the twisted Fock modules. The thesis is well written and contains all the needed material, constructions and proofs.

The thesis consists of three chapters and contains introduction and conclusion parts. The introduction describes the subject of the thesis. Roman provides the motivation, collects the main objects of the study and formulates the main results.

The first chapter is devoted to the special case when one of the parameters of the quantum toroidal algebra is equal to 1 (the Schur specialization case). The chapter contains explicit constructions of the twisted Fock module (depending on the parameters u,n and n'). The action is given in terms of certain vertex operators. The q-W-algebras are considered and the Whittaker vectors are computed.

The second chapter is devoted to the explicit construction of the action of the deformed Virasoro algebras (both twisted and untwisted versions) on the integrable level one representations of the quantum affine sl_2 algebra. The action is constructed via the explicitly written vertex operators. The R-matrix relations are presented, generalizing the Jimbo-Miwa construction

The last chapter describes the link between the representation theory of the quantum toroidal gl_1 and the Double Affine Hecke algebra. Gonin presents a construction of twisted Cherednik

representation of DAHA. Then the author describes the semi-infinite construction of the twisted Fock modules of the quantum toroidal algebra via the level one representations of the quantum affine gl_n algebra. The action of the Chevalley generators are explicitly given in terms of certain vertex operators.

The thesis is concluded with the brief summary of the results .

Summarizing, the thesis of Roman Gonin contains new, interesting and important results of the highest scientific significance. The thesis is based on two publications in well-known peer-reviewed reputable journals. The summary fully and correctly presents the main text. I have no doubt that the Roman Gonin thesis "Twisted representations of toroidal algebras and their applications" fully meets the criteria for a Ph.D. degree. I recommend without any reservations that the candidate be awarded the academic degree of Ph.D. in mathematics

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