Report on the "Candidate of Sciences" thesis "Twisted representations of toroidal algebras and their applications" by Roman Gonin

Representations of toroidal algebras belong now to the mostly intensively developing area in representation theory. Moreover, there are many signs, that their twisted representations will be especially important for constructing generating functions for some important problems in mathematical physics, like it happened earlier for more common infinite-dimentional algebras.

R. Gonin's thesis

The thesis contains three chapters. The first chapter deals with a particular case $q = t$ (in Macdonald's notation). In this case, the twisted Fock module of quantum toroidal gl_1 is constructed in terms of free fermions. Then Roman proves connection of twisted representations with twisted toroidal W-algebras. Also, Gonin applies his results to construction of Nekrasov partition function – a fundamental object in modern approach to supersymmetric gauge theories. Namely, already well-known conjecture states that a particular series of q-deformed Nekrasov's partition functions can be combined into q-isomonodromic tau function, and Roman proves this conjecture in a particular case (the case of algebraic solution).

In the second chapter of the thesis, Gonin constructs representations of twisted and non-twisted deformed Virasoro algebra. These algebras act on the integrable level one representations of quantum affine sl_2. The actions are expressed in terms of the corresponding vertex operators (defined by the intertwining properties). This also gives a bosonization of these algebras.

The third chapter is devoted to the construction of the twisted Fock module for generic parameters of the deformation (q and t) and generic twist (n and n'). Roman constructs the twisted Cherednik representation for Double Affine Hecke algebra. Then Gonin presents the twisted Fock module via semi-infinite construction. The corresponding semi-infinite power is actually an integrable level one representation of quantum affine gl_n. The action of toroidal gl_1 is

expressed via the corresponding vertex operators. The obtained representation is isomorphic to twisted Fock representation.
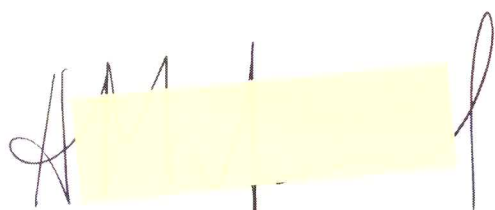
The main results of R.Gonin's thesis

- Construction of twisted Fock module of toroidal gl_1. The algebra acts on an integrable level one representation of quantum affine gl_n. The action is expressed via the vertex operators.

- Bosonization of twisted and non-twisted Virasoro algebra. These algebras act on an integrable level one representation of quantum affine sl_2. The action is also expressed in terms of vertex operators.

- Proof of yhe relation between q-isomonodromic tau function and Nekrasov's partition functions in the case of algebraic solution.

Summary

The thesis results are novel and the candidate demonstrates a solid understanding of various mathematical concepts, especially in the field of representation theory. Simultaneously, the results of the thesis are of fundamental importance since they contain a mathematical proof of one of the most important conjectures in the area.

I have no doubt that the author deserves to be awarded the degree of the Candidate of Sciences in mathematics.

Yours Sincerely,

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Andrei Marshakov
Professor of CAS Skoltech and
Mathematics Department at HSE University,
Moscow, Russia
andrei.marshakov@gmail.com, mars@itep.ru
https://faculty.skoltech.ru/people/andreimarshakov