Skoltech
Skolkovo Institute of Science and Technology

Jury Member Report – Doctor of Philosophy thesis.

Name of Candidate: Saeed Osat

PhD Program: CDSE

Title of thesis: PERCOLATION ON COMPLEX NETWORKS AND ITS APPLICATIONS

Supervisor: Vladimir V. Palyulin, Skoltech

Name of the Reviewer: Nikolai Brilliantov

I confirm the absence of any conflict of interest	**Signature:**  **Date: 24-09-2021**

Reviewer's Report

The work "PERCOLATION ON COMPLEX NETWORKS AND ITS APPLICATIONS" submitted as a PhD thesis by Saeed Osat studies an important novel type of complex networks called multiplex networks. The thesis is submitted in a form of a coherent academic treatise consists of 8 chapters and is based on 7 papers published in scientific journals. Among them there are 3 first author papers and 4 second author papers. 5 articles are published in Q1/Q2 journals indexed in WOS/Scopus with 2 among them counted in the Nature Index. This securely fulfills the requirements of the PhD thesis defense policy of Skoltech. The first out of the 8 chapters is an introduction. The following 6 chapters consist each of a text of the corresponding paper and a short description of the contribution of the applicant to the published work. The final chapter summarizes the scientific contributions of the thesis.

Overall, I am very satisfied by the scope and the comprehensiveness of the study of the relatively new class of complex networks. The model of multiplex networks is a very useful approach for the description of complex transportation systems, www networks, social networks and others. The practical importance of obtained results is clearly shown in the text. The treatise presents a significant advancement in the field.

The high-quality papers support this statement and show the author's ability to utilize state-of-the-art computational methods as well as deep knowledge of the relevant literature.

The first part of the introduction covers the basics of the classical network theory as well the generalization of definitions and notions such as k-core, percolation, giant component, community structure and geometric correlations. It also introduces the new notions which are specific for the multiplex case including interlayer degree correlation and the edge overlap. The second part of the chapter describes the contents of the papers attached together as a treatise and binds them together in a cohesive form.

The second chapter considers a simple model of neuronal dynamics on networks to show that the degree distribution of the underlying neuronal network may trigger power-law distributions for neuronal avalanches. Contrary to the previous opinion in the literature, this could happen even in the case when the system is not in a critical regime.

The third chapter generalizes the notion of k-core used in simple networks to the multiplex networks and is called Gk-core (generalized k-core). The analytical and numerical approaches for Gk-core percolation were applied to real-world networks such as transcription networks, yeast-protein networks, computer networks etc.

The fourth chapter is devoted to observability transition and generalization of the notion of observability to multiplex networks. Again, the approach was tested on the real-world networks (US air transportation multiplex network, C. Elegans connectome, Arxiv, etc.).

Chapter five addresses the embedding of multiplex networks into a hyperbolic space. The correspondence of community structure and hyperbolic embedding is shown. Then the correlations between community structure between the layers and the similarity of the nodes in the embedding is revealed.

Chapter six culminates in consideration of a very physically important question, namely, of an optimal percolation in multiplex networks. This part of work pioneers this question in the field of multiplex networks and discovers its essential difference from a simple network case. The main conclusion states that an attempt of reducing of a multiplex network to a monoplex case leads to the substantial error in identification of the set of structural nodes, i.e. neglecting of the multiplex structure may effect in a significant inaccuracy in the estimation of its robustness.

The currently final chapter with the results, chapter 7, deals with the k-core structure of real multiplex networks (internet, ArXiv coauthorship network etc.). The main outcome is that for the cases with heterogeneous degree distribution of the network a strong k-core structure is well predicted by positive degree-degree correlations. In the other case of homogeneous networks the strong k-core structure appears due to positive correlations at the level of node similarities.

While the general impression from the thesis is excellent there are a few questions/remarks to be mentioned. Some of them stem from my curiosity and the interest to the problem and may be rather general.

1) Although the authors mentioned a few times Hamiltonian of a network he does not explain its meaning in the current context. I believe that it would be worth to add somewhere a discussion about this notion. The related question – does there exist any relation/similarity between the cost function and the partition function? I believe that as far as the Hamiltonian may be introduced (as well as effective temperature) in the context of networks, one can define the partition function. Has this been done?

2) General question: You did not discuss the notion of entropy of a network, does it make sense? Will the entropy of a multiplex network be additive with respect to the partial entropies of the layers?
3) In the case of percolation in a multiplex network can two (or more) percolating clusters coexist there? It is believed that this is not possible in the case of a common (one-layer) network.
4) Is it possible that a giant percolating cluster is comprised partly of nodes of a first layer and partly of nodes of the second layer of a multiplex network, while there are no percolating clusters separately in either layers?
5) The last of the published papers on the list is neither a part of the introductory chapter nor it was added as a separate chapter at the end. I suggest to fix this by adding the corresponding parts.
6) Apart from this I have a list of minor corrections:

- There are a few typos across the manuscript to be taken care of.
- Everywhere across the text: The shortening "Fig" should be followed by the full stop, i.e. "Fig."
- Page 4 in the definition of the clustering coefficient there should be a comma after the formula and before the word "where"
- Page 43. The sentence "As clear from the figure, randomization weakens the k-core spectrum." I would recommend to specify the number of the figure and add a definite article in from of "randomization"
- Page 47. "Writing of the manuscript was mostly done by the main Author Ali Faqeeh. All the authors contributed in the editing of the paper." The word author in the first sentence should be decapitalized and "contributed in" changed to "contributed to"

Provisional Recommendation

☒ *I recommend that the candidate should defend the thesis by means of a formal thesis defense*

☐ ~~*I recommend that the candidate should defend the thesis by means of a formal thesis defense only after appropriate changes would be introduced in candidate's thesis according to the recommendations of the present report*~~

☐ ~~*The thesis is not acceptable and I recommend that the candidate be exempt from the formal thesis defense*~~