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Jury Member Report – Doctor of Philosophy thesis.

Name of Candidate: Saeed Osat

PhD Program: CDSE

Title of thesis: PERCOLATION ON COMPLEX NETWORKS AND ITS APPLICATIONS

Supervisor: Vladimir V. Palyulin, Skoltech

Name of the Reviewer: Pavel Krapivsky

I confirm the absence of any conflict of interest	**Signature:**  **Date: 23-09-2021**

Reviewer's Report

The thesis work of Saeed Osad is devoted to the study of percolation on complex networks, observability of transitions in multiplex networks, core percolation, and k−core structure of multiplex networks. The thesis starts with a short outline and a list of publications and continues (Chap. 1) with an excellent general introduction to the science of complex networks, percolation, the notions of core percolation, and the k−core structure of networks. In this chapter, Saeed Osad also describes the main results of the published papers. The following Chapters 2–7 are the copies of the published papers. Chapter 8 gives brief conclusions.

The introductory chapter 1 introduces the basic objects and concepts. Some are classical in graph theory, others are recent (multiplex networks, etc.). One beautiful concept which is almost classical, but somehow little known, is the concept of k−core and the related concept of the core. I've never heard about the latter although it is classical going back to the work of Karp and Sipser on the pruning procedure called greedy leaf removal. The giant core in a random graph emerges substantially later than the percolation transition; both transitions are continuous. An interesting generalization of the pruning transition gives birth to a giant Gk−core. All these phase transitions are very nice, and this part of the work is particularly interesting in my eye. (Admittedly, as it is mostly scientifically intriguing and perhaps a bit can be

done analytically.)

As in most work on networks, Erdős and Rényi are the first scientists mentioned. They wrote outstanding papers that generated great interest, but it is unfair to ignore e.g. Paul Flory who was a true pioneer and had tons of results (the cluster size distribution, the understanding of the percolation transition, etc.) much earlier than anyone else. His work was phrased using terminology from polymer science and it may be difficult to see, especially to mathematicians and computer scientists, that he was doing essentially the same. Anyway, here are a few relevant studies: P. J. Flory, J. Amer. Chem. Soc. 63, 3083 (1941); W. H. Stockmayer, J. Chem. Phys. 11, 45 (1943); P.J.Flory, Principles of Polymer Chemistry (Cornell University Press, Ithaca, 1953). I am mentioning this not just to be fair but also since a kinetic approach used in that work is overwhelmed by the probabilistic approach to random graphs frequented by mathematicians and computer scientists. Also, the original paper by Erdős and Rényi was called “Evolution of random graphs”, so they thought about the process dynamically. The percolation transition in a random graph is much easier to understand, at least in my view, using the kinetic approach. So I wonder would it be possible to advance the understanding of the phase transitions related to the birth of the giant core, and more generally the giant Gk−core, using the kinetic approach? I am sure it is not a straightforward generalization of the description of the cluster size distribution, and the emergence of the giant component, etc. in the evolving random graph. So it is in principle feasible that the kinetic approach cannot handle such quantities, but it is worth trying.

Another thought that has come to my mind is the concept of onion decomposition. This recent concept is very much related to the k−core decomposition. Thinking about it one starts to appreciate not only the importance of phase transitions that occur when the giant k−core, the giant Gk−core, etc. emerge in a random graph, but the concepts of k−shells, the onion decomposition, and the onion layers. All those quantities can be defined for an arbitrary graph. It seems that Saeed Osad is well equipped for extending the tools he used in the PhD work to those other related and intriguing quantities.

Overall, the thesis by Saeed Osad reports high-quality research. The author demonstrated originality and creativity, he established intriguing results using advanced numerical techniques, and he demonstrated theoretical strength and excellent knowledge of various branches of network science. I do not personally know Saeed Osad, but from the thesis and published papers he appears to be a mature and talented scientist and there is no question in my mind that the defense of this thesis could be made in Skolkovo Institute of Science and Technology.

Provisional Recommendation

☒ *I recommend that the candidate should defend the thesis by means of a formal thesis defense*

☐ ~~*I recommend that the candidate should defend the thesis by means of a formal thesis defense only after appropriate changes would be introduced in candidate’s*~~

~~*thesis according to the recommendations of the present report*~~
☐ ~~*The thesis is not acceptable and I recommend that the candidate be exempt from the formal thesis defense*~~