Report on the thesis “Integrable structure of affine Yangian“ by Ilia Vilkovisky

The thesis is partly mathematical and partly related to physics. Mathematical part is closer to me so let me write few words about the main results. Integrable systems – the old but still very popular object. Note that theory of quantum groups arise from integrable systems. Anyway now quantum groups is the main tool in studying such systems – more than that ,quantum groups give us the way to construct big systems of commuting operators and find eigen vector eigen values by Bethe anzats. Recently new class of algebras appear – toroidal algebras and their degenerations – affine Yangians. Such algebras naturally appear when you study KdV – type systems .They are the set of Hamiltonians acting in the representations of W- algebras. And toroidal -like algebras can be understood as trigonometric deformation of W- algebras. Such deformations sometimes calling“elliptic” W- algebras.

There are two cases – W algebras related with full linear group and with orthogonal group. Methods are similar – in first case we can use the toroidal algebra in LLR – realization and in orthogonal case the K- matrix method developed by Sklyanin. Note that W- algebra appear naturally in 4- dimensional topological theory and related studying of instanton manifolds. Such W – algebras become very popular and any information about integrable systems is using by many specialists. Now about Ilia results.

1. Starting with RLL relations he found the generators and relations for affine Yangians.

2. He manage to describe orthogonal W- algebra in terms of K- matrix, find generators and relations.

3. In the orthogonal case he finds the explicit formulas for the Hamiltonians In all cases Ilia wrote Bethe equations.

Note that he developed the formalism of reflection matrices and wrote the KZ- type of integral of motions. The reflection machinery - very interesting way to study integral of motions.

I think the Ilia thesis is outstanding work – many new nontrivial ideas and technical results. He is one of the best young mathematical physicists and I expect a lot – he definitely become the leading specialist in the integrable systems

I strongly recommend the candidate be awarded the degree PhD in mathematics.

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