

Report on the thesis  
«Integrable structures of the affine Yangian»,  
by Ilia Vilkovisky

The thesis is devoted to integrable structures that appear in two-dimensional conformal field theory. The study in this field began with the seminal series of papers of Bazhanov, Lukyanov and Zamolodchikov devoted to quantum KDV integrable system. Big progress appears since the discovery of Ordinary Differential Equation/Integrable Model (ODE/IM) correspondence by Dorey and Tateo. Using this approach certain system of algebraic equations (Bethe equations) describing the spectrum of the commuting Hamiltonians was found.

Then, huge progress was achieved due to the relation of these integrable systems to supersymmetric gauge theories and geometric representation theory. The main works are due to Nekrasov, Shatashvili, Okounkov, Aganagic, Maulik, Smirnov. In particular, it was observed that usually there is another point of view on the CFT (especially  $W$ -algebra) symmetry through so-called affine Yangians. Moreover, another set of Bethe equations for one-parameter deformation of the integrable system was proposed and proven, first using geometric methods, and then (in  $q$ -deformed setting) by Feigin, Jimbo, Miwa, Mukhin by purely algebraic methods. In the thesis, Ilia Vilkovisky suggested new transparent proof of these Bethe equations and provide vast generalizations which are not available by other methods.

The method of the thesis lies in the spirit of the quantum inverse scattering method. It starts from the infinite-dimensional  $R$  matrix which is essentially the Liouville reflection matrix. In geometric terms it is Maulik-Okounkov instanton  $R$  matrix. Starting from it the RLL Yangian was constructed, it appears that it is closely related to affine Yangian of  $\mathfrak{gl}_1$ . The affine  $W$  algebras of type  $A$  can be obtained as certain quotients of this Yangian. Using this presentation the Bethe equations were proven. Very elegant approach, based on KZ equations was used in the proof.

The main results of the paper are the following

- Relation of the RLL presentation and current presentation of the affine Yangian of  $\mathfrak{gl}_1$  was shown.
- Bethe equations for integrable systems for integrable system corresponding to affine  $W$  algebra of type  $A$  were proven.
- The  $K$  matrices satisfying Skyanin  $KRK\bar{R}$  relation were constructed for the Maulik-Okounkov instanton  $R$  matrix. The corresponding Bethe equations were proven.
- $q$  deformation of  $K$  matrices were constructed. The corresponding  $W$  algebras were studied, the formulas for integrals of motion were found.

The thesis consist of introduction and three chapters. In the introduction the motivation, main problems and the results were explained. Three chapters corresponds to three papers on which the thesis is based on. I find results on any of this chapters remarkable, the difficult problems were solved brilliantly. The results are published in good journals and well known to experts. The text is well written.

I strongly recommend that the candidate be awarded the academic degree Ph.D. in mathematics.

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