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Report on the Doctor of Philosophy in Mathematics thesis
"Integrable structures of the affine Yangian"
by Ilya Vilkoviskiy.

Ilya Vilkoviskiy's thesis deals with integrable structures in conformal field theories. This thesis developed an approach based on the symmetry of the affine Yangian. This approach is closely related to the quantum inverse scattering method. This allows one to consider a wide class of integrable systems as different representations of the same Yang–Baxter algebra. In this case, the spectrum of integrals of motion is expressed in terms of the roots of the Bethe equations. In fact, this research is the development of mathematical tools necessary for studying of quantum integrable systems. The latter are used in the supersymmetric gauge theories. They are also used to describe strongly correlated systems. Recently, these systems have been experimentally implemented in quasi-one-dimensional crystals and systems of ultracold atoms. Thus, the relevance of the study is beyond doubt.

The main text of the thesis is presented in four chapters. The first chapter is essentially an introduction. It briefly summarizes the content of the thesis, introduces the basic concepts and notation.

In Chapter 2, the author studies integrable structures related to $\widehat{\mathfrak{gl}}(1)$ affine Yangian. The main objects of study are the Maulik–Okounkov R-matrix, which plays the role of the Liouville reflection operator. The author discusses in detail the current realization of the corresponding RLL-algebra, constructs local integrals of motion for W-algebras of the A-series, and obtains the Bethe equations describing the spectrum of integrals of motion. It is interesting to note that the obtained Bethe equations are similar to those arising from the quantization of the KP equation. The author also shows that the central elements of $YB(\widehat{\mathfrak{gl}}(1))$ correspond to the singular vectors of the algebra W_n in the tensor product of n Fock spaces. Using the technique developed in the first chapter, the author calculates the norms of the on-shell Bethe vectors.

In the third chapter of the thesis, the author generalizes the results and methods of Chapter 2 to W-algebras of types B, C, and D. Here, the author introduces an analog of the Sklyanin K-matrix, which satisfies the reflection equation and is used to study quantum integrable systems with open boundaries. In this case, the boundary corresponds to the ends of the affine Dynkin diagram for the given integrable system. Three solutions to the reflection equation are constructed. The author also obtains explicit formulas for off-shell Bethe vectors. It is shown that if the parameters on which the off-shell Bethe vector depends satisfy the system of Bethe equations, then the KZ operator acts diagonally on this vector.

Chapter 4 is devoted to studying of deformed W-algebras associated with Lie algebras of B, C, D types. The author proposes a new and fairly simple method based on the use of screenings. Within the framework of this approach, it is possible to construct q-deformed

integrals of motion of arbitrarily high spin for deformed W-algebras, which commute with an additional affine screening charge. The constructed integrals of motion turn out to be an elliptic generalization of the ones in non-deformed algebras. The author also finds explicit formulas for the deformed R-matrix and the Sklyanin reflection operators.

I now give a list of the main results obtained in the thesis.

- An explicit current description of the RLL-algebra corresponding to the Maulik–Okounkov R-matrix is given.
- Three solutions to the reflection equation corresponding to the Maulik–Okounkov R-matrix are found.
- Explicit formulas are obtained for the integrals of motion of q-deformed W-algebras of B, C, and D types.
- The R-matrix and Sklyanin K-operator corresponding to q-deformed W-algebras are found.

In summary, Ilya Vilkoviskiy's thesis shows that affine Yangian integrable structures naturally appear in the context of conformal field theories. This observation opens up new possibilities for studying these theories. The results of the thesis are certainly new. The candidate shows a deep understanding of the problem under study and related issues.

I have no doubts that this thesis satisfies all the requirements for the degree of the Doctor of Philosophy in Mathematics.

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