Skoltech
Skolkovo Institute of Science and Technology

Jury Member Report – Doctor of Philosophy thesis.

Name of Candidate: Ivan Sechin

PhD Program: Mathematics and Mechanics

Title of Thesis: Quantum R-matrix identities and integrable systems

Supervisor: Professor Anton Zabrodin

Co-supervisor: Dr. Andrei Zotov, Steklov Mathematical Institute, RAS

Name of the Reviewer: Bershtein Mikhail

I confirm the absence of any conflict of interest (Alternatively, Reviewer can formulate a possible conflict)	30-09-2022 **Date: DD-MM-YYYY**

The purpose of this report is to obtain an independent review from the members of PhD defense Jury before the thesis defense. The members of PhD defense Jury are asked to submit signed copy of the report at least 30 days prior the thesis defense. The Reviewers are asked to bring a copy of the completed report to the thesis defense and to discuss the contents of each report with each other before the thesis defense.

If the reviewers have any queries about the thesis which they wish to raise in advance, please contact the Chair of the Jury.

Reviewer's Report

Reviewers report should contain the following items:

- Brief evaluation of the thesis quality and overall structure of the dissertation.
- The relevance of the topic of dissertation work to its actual content
- The relevance of the methods used in the dissertation
- The scientific significance of the results obtained and their compliance with the international level and current state of the art
- The relevance of the obtained results to applications (if applicable)
- The quality of publications

The summary of issues to be addressed before/during the thesis defense

The thesis is devoted to the theory of integrable systems. During the last 50 years, there are many such

systems were found, probably the most know are named after Calogero, Toda, Ruijsenaars, Macodnald. In the thesis new examples of integrable systems were constructed. These systems have NM×NM matrix structure. They are common generalizations of two known integrable systems which appear in cases N=1 and M=1.

The main results of the paper are the following

- The common generalization of integrable Euler-Arnold tops and spin Calogero system was constructed. The construction works for all known classes of Calogero integrable systems: rational, trigonometric and elliptic.
- A relativistic version of the system above was constructed. This system is a common generalization of spin Ruijsenaars–Schneider model and relativistic tops.
- The new quadratic algebra RLL algebra was found. This algebra is a quantization of the Poisson bracket used in the model above. The corresponding R matrix is a common generalization of the Baxter-Belavin elliptic R matrix and Felder dynamical R matrix. The relations generalize Sklyanin algebra relations and elliptic quantum group relations.

The methods are based on the machinery in the theory of integrable systems: Lax matrices, classical and quantum R-matrices. The key tool is an Associative Yang-Baxter equation which can be viewed as a matrix analog of the Fay identity. This is a new and interesting application of the Associative Yang-Baxter equation which is known for like 40 years.

The thesis consists of an introduction and three chapters. In the introduction the Associative Yang-Baxter equation and results of the thesis were explained. Three chapters correspond to three papers on which the thesis is based. The results a truly remarkable. The results are published in good journals and are well known to experts. The text is well written, I spotted only minor typos.

I strongly recommend that the candidate be awarded the academic degree Ph.D. in mathematics.

Provisional Recommendation

☑ *I recommend that the candidate should defend the thesis by means of a formal thesis defense*

☐ *I recommend that the candidate should defend the thesis by means of a formal thesis defense only after appropriate changes would be introduced in candidate's thesis according to the recommendations of the present report*

☐ *The thesis is not acceptable and I recommend that the candidate be exempt from the formal thesis defense*