

Jury Member Report – Doctor of Philosophy thesis.

Name of Candidate: Ivan Sechin

PhD Program: Mathematics and Mechanics

Title of Thesis: Quantum R-matrix identities and integrable systems

Supervisor: Professor Anton Zabrodin

Co-supervisor: Dr. Andrei Zotov, Steklov Mathematical Institute, RAS

Name of the Reviewer: Andrei Pogrebkov

I confirm the absence of any conflict of interest	
(Alternatively, Reviewer can formulate a possible conflict)	Date: 29-09-2022

The purpose of this report is to obtain an independent review from the members of PhD defense Jury before the thesis defense. The members of PhD defense Jury are asked to submit signed copy of the report at least 30 days prior the thesis defense. The Reviewers are asked to bring a copy of the completed report to the thesis defense and to discuss the contents of each report with each other before the thesis defense.

If the reviewers have any queries about the thesis which they wish to raise in advance, please contact the Chair of the Jury.

Reviewer's Report

Reviewers report should contain the following items:

- Brief evaluation of the thesis quality and overall structure of the dissertation.
- The relevance of the topic of dissertation work to its actual content
- The relevance of the methods used in the dissertation
- The scientific significance of the results obtained and their compliance with the international level and current state of the art
- The relevance of the obtained results to applications (if applicable)
- The quality of publications

The summary of issues to be addressed before/during the thesis defense

Brief evaluation of the thesis quality and overall structure of the dissertation.

Dissertation of Ivan Sechin consists of Introduction, four chapters, three appendices, concluding remarks and list of references. Introduction gives short description of the identities on quantum R-matrices and their correspondence to some functional relations. A brief overview of the results submitted for defence is also given here. Thesis quality is very high.

The main part of the thesis consists of four works, where I. Sechin was one of coauthors. Introduction gives a list of results, presented for defense, together with perspectives of their applications and future development. List of references contains 92 titles.

The relevance of the topic of dissertation work to its actual content

The thesis of Ivan Sechin devoted to an extremely relevant topic: construction and research of new completely integrable systems of a finite number of degrees of freedom. This kind of research describes the remarkable properties of such systems and arouses great interest among specialists. Content of dissertation is in exact correspondence to its announced subject.

The relevance of the methods used in the dissertation

The main distinctive peculiarity of the thesis is unification of the extensions of the spin Ruijsenaars-Schneider particle systems by integrable systems of tops. Thus, the Chapter 2 is devoted to non-relativistic generalized interacting tops. Interaction of spin variables is determined by the quantum R-matrices, the Hamiltonian and Lax operators of this system are expressed through them. The author describes the Hamiltonian structure of the generalized interacting integrable system of tops and shows, that the Lax operator for the system has the modified dynamical classical r-matrix structure. By these means he proves the commutativity of the Hamiltonians of this system and derives corresponding constraints.

In the Chapter 3 author introduces relativistic version of the generalized interacting integrable system of tops. Author introduces systems that unify extensions of the spin Ruijsenaars-Schneider particle systems and the relativistic integrable tops. Their description is given by means of the introduced Lax pairs with a spectral parameter. These Lax pairs include the quantum R-matrix, that satisfy the set of quantum R-matrix identities, derived by the author. The Lax equations results in the equation of motion on a set of constraints of coordinates.

Finally, the Chapter 4 deals with the quantum R-matrix version for the generalized interacting system of tops. As initial point of quantization the author chooses the modified dynamical classical r-matrix structure, that was introduced in the previous chapters. Specific property of the quantum R-matrix is its dependence on space coordinates only. It has a form of a $GL(M)$ block Felder dynamical R-matrix with nondynamical $GL(N)$ quantum R-matrices inside the blocks. By these means an L-operator and RLL-algebra corresponding to this quantum R-matrix are introduced. The author also defines the quadratic algebras that are given via generators and relations, equivalent to the RLL-algebras in the case of elliptic Baxter-Belavin R-matrix in the blocks. The author introduces generators of this algebra that satisfy the Sklyanin algebra relations. Besides this, some additional relations on the operators that generalize the relations in the small elliptic dynamical quantum group are given.

The scientific significance of the results obtained and their compliance with the international level and current state of the art

Investigation performed in the thesis of I. Sechin is based on the observation that system of interacting

integrable tops can be considered as the extension of the spin Calogero–Moser system of interacting particles and the extension of the integrable top of the Euler–Arnold type.

Main results derived in dissertation are:

- a) the explicit expression for the $gl_{\{NM\}}$ -valued Lax pair with spectral parameter for the generalized interacting integrable system of tops and the Hamiltonian description of this model, including the classical dynamical r -matrix structure associated with the constructed Lax pairs.
- b) independence of the construction on the explicit form of the R -matrix;
- c) the description of the Lax structure of a generalized model of relativistic interacting tops, constructed by means of the quantum R -matrix satisfying the associative Yang–Baxter equation;
- d) construction of $GL(N, M)$ models that generalizes the classical spin Ruijsenaars–Schneider systems and the relativistic integrable tops on $GL(N)$ Lie group;
- e) construction of the generalized quantum $GL(N, M)$ dynamical R -matrix by means of the $GL(N)$ solution of the associative Yang–Baxter equation;
- f) derivation of the quadratic quantum algebra constructed on the base of the dynamical RLL-relation, which corresponds to the elliptic version of R -matrices introduced in the thesis.

All the above results are at the top of scientific publications and have no analogs in the world literature.

The relevance of the obtained results to applications (if applicable)

Dissertation of I. Sechin has theoretical character. In this sense its main result consists in development of methods of investigation of classical and quantum integrable models with finite number of degrees of freedom.

The quality of publications

The thesis is based on four articles published in journals of categories Q1 and Q2, and citation bases WoS and Scopus. The English language of the presentation could be improved, while this remark does not diminish quality of thesis.

Provisional Recommendation

☒ *I recommend that the candidate should defend the thesis by means of a formal thesis defense*

☐ *I recommend that the candidate should defend the thesis by means of a formal thesis defense only after appropriate changes would be introduced in candidate's thesis according to the recommendations of the present report*

☐ *The thesis is not acceptable and I recommend that the candidate be exempt from the formal thesis defense*