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Skolkovo Institute of Science and Technology

Jury Member Report – Doctor of Philosophy thesis.

Name of Candidate: Ivan Sechin

PhD Program: Mathematics and Mechanics

Title of Thesis: Quantum R-matrix identities and integrable systems

Supervisor: Professor Anton Zabrodin

Co-supervisor: Dr. Andrei Zotov, Steklov Mathematical Institute, RAS

Name of the Reviewer: Vladimir Roubtsov

I confirm the absence of any conflict of interest I confirm the absence of (Alternatively, Reviewer can formulate a possible conflict) any conflict of interest.	**17/10/2022**

The purpose of this report is to obtain an independent review from the members of PhD defense Jury before the thesis defense. The members of PhD defense Jury are asked to submit signed copy of the report at least 30 days prior the thesis defense. The Reviewers are asked to bring a copy of the completed report to the thesis defense and to discuss the contents of each report with each other before the thesis defense.

If the reviewers have any queries about the thesis which they wish to raise in advance, please contact the

Reviewer's Report

Reviewers report should contain the following items:

- Brief evaluation of the thesis quality and overall structure of the dissertation.
- The relevance of the topic of dissertation work to its actual content
- The relevance of the methods used in the dissertation
- The scientific significance of the results obtained and their compliance with the international level and current state of the art
- The relevance of the obtained results to applications (if applicable)
- The quality of publications

The summary of issues to be addressed before/during the thesis defense

Provisional Recommendation
I recommend that the candidate should defend the thesis by means of a formal thesis defense

17/10/2022

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LABORATOIRE ANGEVIN DE RECHERCHE EN MATHÉMATIQUES – UMR CNRS 6093
FACULTÉ DES SCIENCES – DÉPARTEMENT DE MATHÉMATIQUES

Angers, 15 October, 2022

Report on the manuscript
«Quantum R-matrix identities and integrable systems»
presented by Ivan Sechin as a PhD Thesis for the purpose of obtaining academic degree Doctor of Philosophy in Mathematics

The manuscript presents in a synthetic form the research work of Ivan Sechin, and more particularly that which he carried out during his doctoral studies.

Ivan Sechin's scientific activity mainly concerns a modern Mathematical Physics subject which is an amalgam of methods in Quantum Groups (QG), Theory of Integrable Systems (IS) and Lie Algebra theory.

The candidate presents a set of results obtained over the past 3 years, as part of a Doctoral Program in Mathematics and Mechanics of the Skolkovo Institute of Science and Technologies.

They are mainly situated in the topic of classical and quantum IS, and aim, on the one hand, to present a complete and rigorous definition and a study for certain generalizations of famous (non-)relativistic many–body Calogero–Moser–Ruijsenaars-Schneider (CM-RS) systems and (in another limiting case) no less famous Euler-Arnold tops. They obeyed both classical and quantum integrability description. It is a result (in classic cases) of a Lax representation constructions as well as from the existence of a dynamical classical r-matrix for the generalized interacting integrable tops in the non-relativistic case. The quantization of this classical r- matrix leads to dynamical QG (=quantum dynamical RLL-algebras). The obtained algebras (in the elliptic case) are quadratic quantum algebras generalizing the famous Sklyanin algebra.

On the other hand, it is a wide range of calculations for certain identities on the quantum R-matrices, mainly, the associative Yang–Baxter equation.

The Thesis is based on three publications (two – in a collaboration with A. Zotov and one – with A. Zotov and A. Grekov) and structurally contains four chapters including the Introduction and the Conclusion).

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The First Chapter (Introduction) contains a brief review of main Thesis "actors" (CM Hamiltonians, Fay identity, AYBE and quantum R-matrices etc.) and a description of main Thesis results Chapter by Chapter.

A review of generalized non-relativistic interacting top–like IS with spin and its quantum R-matrices, Lax and Hamiltonian representation are discussed in the Chapter 2 of the Thesis. These results are heavily based on the paper of the Candidate (together with A. Grekov and A. Zotov) in JHEP 2019, no. 10, 081. They introduce a family of classical IS describing dynamics of M interacting gl_N integrable tops extending the previously known model of interacting elliptic tops. The author's construction is based on the GL_N -matrix satisfying the AYBE. The obtained systems can be considered as extensions of the spin type CM system with (the classical analogues of) anisotropic spin exchange operators given in terms of the R-matrix data. In the limiting case when $N = 1$the spin CM model is reproduced and in the case $M = 1$ case - the Euler–Arnold integrable top. Explicit expressions for the Lie algebra gl_{MN} -valued Lax pair with spectral parameter and its classical dynamical r-matrix are obtained together with the corresponding Hamiltonian structure.

The third Chapter describes with great care and precision the construction of the relativistic version for the previously studied interacted tops: simultaneous extensions of the spin RS particle systems and the relativistic integrable tops. These results are heavily based on the paper of the Candidate (together with A. Zotov in Theop. Math. Phys., 205:1 (2020) 1292–1303 , arXiv: 2011.09599).

Analogously to the previous Chaper (non-relativistic case) the Candidate describes a family of integrable GL_{MN} models which generalize classical spin RS systems (the case $N = 1$) on one limiting case and relativistic integrable tops on the Lie group GL_N (the case $M = 1$) on the other side. The described models are obtained using the Lax pair with a spectral parameter. The equations of motion are also obtained. The Lax representation construction uses the GL_N R-matrix in the GL_Nfundamental representation.

The Chaper 4 is devoted to a «quantization"of the systems described in the Chapters 2 and 3. This Chapter also can be «splitted"on the quantization of the non-relativistic classical models of the Chapter 2 and the corresponding quantization of the relativistic models from the Chapter 3.

Both quantizations are based on R-matrix technique, where corresponding R-matrix of the first part is a solution of dynamical QYBE (which has GL_M-valued Felder's dynamical R-matrix for $N = 1$ and a version of the Baxter–Belavin's elliptic R-matrix for $M = 1$).

In the second part of the Chapter a quadratic dynamical RLL-type algebra is discussed. This algebra are defined by a quantum R–matrix which is related to the SL_{NM} –bundles over the elliptic curve and contains as a generalization simultaneously the elliptic non–dynamical Baxter– Belavin and dynamical Felder R–matrices. This quadratic algebra generalizes both the Sklyanin algebra relations and the Felder–Tarasov–Varchenko elliptic QG, coinsiding with them in the particular cases M = 1 and N = 1 respectively.

It worse to remark that the results of this Chapter are heavily based on two papers of Candidate (both with A. Zotov) published in Russian Math. Surveys, 74:4 (2019) 767–769; arXiv: 1905.08724 and in Theor. Math. Phys., 208:2 (2021), 1156–1164, arXiv: 2104.04963.

Concluding I want to remark that I. Sechin has proposed in this Thesis very interesting R-matrix identities which are based on various analogues of Fay identities and interpretations of AYBE solutions. These fairly developed constructions have been written with care and an eye for detail and pedagogy which greatly facilitates the reading of the associated above-mentioned articles.

Summarizing my feeling from reading the Thesis and articles of the applicant, I have no doubt about the high scientific level of both manuscript and the articles themselves, the structures and results presented in them. They, in my opinion, relate to topical and interesting problems of modern theory of IS and quantum algebras. It emerges that I. Sechin participated in a quite significant way in works of major importance in the domain of classical and quantum IS which were started by M. A. Olshanetsky and his collaborators. He has also acquired an excellent mastery of the theoretical bases and techniques required in the theory of IS and quantum algebras, and has already shown his ability to develop original research of high quality.

All theorems, statements and conclusions are well-founded. Their approbation is guaranteed by high scientific criteria of the editions in which they are published. Few critical points:

a) The Thesis proposes some interesting interpretations of the Fay identity and related solutions of AYBE. Studying both classical and quantum application the Candidate did not mention and did not discuss an interesting «classical–quantum dichotomy of AYBE» when the same associative R-matrix can be interpreted both as a «classical» and a «quantum» objects;

b) The thesis is written heavily on the material of three papers in a collaboration. It would be useful to indicate explicitly the Candidate's personal input.

These points are by no means essential.

For these reasons, I agree without reservations to the defense of the thesis.
I recommend that the candidate should defend the thesis by means of a formal thesis defense.

Comparing his Thesis with known to me PhD works in France, UK, Norway and Netherlands I should evaluate it like a similar to be "very good" (= « très honorable» in France).

Thus, the presented work meets all the requirements for PhD dissertations. The author certainly deserves this degree.

Professeur Vladimir Roubtsov,

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