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# Thesis Changes Log

**Name of Candidate:** Vadim Prokofev

**PhD Program:** Mathematics and Mechanics

**Title of Thesis:** Integrable hierarchies of nonlinear differential equations and many-body systems

**Supervisor:** Prof. Anton Zabrodin

*The thesis document includes the following changes in answer to the external review process.*

The integrals $I_m$ for the elliptic CM system can be found from the Lax representation (without spectral parameter) found by Calogero [1975], so there is no need here to use d'Hoker-Phong results
**Reviewer** Prof. Pavel Etingof

**1. Formula (1.24) contains a misprint.**
**2. Is the coefficient 2 for the potential needed in (1.24)? Of course, for classical systems it does not matter since the coupling constant can be renormalized, but I believe the standard normalization is without 2. Same comment for (1.17).**
**3. Chapter 3, line 3,4: the label 4.3 repeats several times in the same sentence, probably it is a misprint**

I've corrected misprints

**4. Formula (3.10). I am not sure what is meant by a "formal Lie algebra". The set defined by (3.10) is not a Lie algebra, some vanishing conditions are needed to make it so (I am sure the author understands this perfectly well, just uses an unusual terminology which may be confusing).**

I've used a notation from the original article of Ueno and Takasaki. As it is written below in the text the product of two matrices is well-defined only in case of their order bounded from below or above, so it shouldn't be confusing.

**Reviewer** Prof. Alexander Veselov

I've corrected all mistakes and misprints mentioned in review

**The equation (1.1) is not equivalent to (1.2) in that form.**

As I defined A_3 below explicit calculation of commutator in r.h.s of (1.2) results in KdV equation (1.1).

**"...Calogero-Moser system (1.17)..." is not introduced yet.**
I mentioned it here in order to only motivate the introduction of more general KP hierarchy.

**"The other objects of study in my thesis is (?) a (?) classical many body systems integrable according to Liouville i.e. contains maximal number of independent integrals of motion." Not quite, condition of the involutivity is missing.**

I added the condition that they commute with each other.

**I do not think that this paper by Calogero and Marchioro actually contains the proof of classical integrability, so I would refer here only to the seminal work of Moser.**

In the paper by Calogero and Marchioro the case of three particles was considered and authors use 3 integrals of motion to solve the system.

**The integrals $I_m$ for the elliptic CM system can be found from the Lax representation (without spectral parameter) found by Calogero [1975], so there is no need here to use d'Hoker-Phong results**

However for elliptic solutions we use a spectral parameter dependence so for explicit calculation of Hamiltonians it is more useful to use d'Hoker-Phong results.

**I believe this was done before in Krichever [1978].**

Krichever's work contains only results for the second and third times.

**The author indeed follows Chapter 7 (not 5 and 6, as the author wrote) of Dickey very closely, maybe too closely for the summary of PhD thesis. It does not look good, when all the theorems in the Summary (Theorems 1, 2 and 3) are borrowed from Dickey's work.**

My motivation was to lighten up some things that were used in papers, but were not proven in it since they are well-known in integrability. I mean the equivalence of pseudodifferential operator approach and Hirota bilinear equations for KP matrix KP and 2d Toda lattice hierarchies. I used Chapters 2 and 3 of my Thesis to do so. Since it is not a main point of my thesis I've followed a well-known source.

**Most importantly, it should be mentioned that the notion of Baker-Akhiezer function was introduced long before 2003, in the most generality - by Krichever [1977]. In the context of tau-function, one should mention also the important work by Segal and Wilson (Mathem. IHES, 63, 1-64, 1985), who interpreted Krichever's construction in terms of Sato's Grassmannian approach.**

In the second Chapter I've moved from historical remarks. I have not mentioned Grassmanians and free fermions also, though they are important for infinite hierarchies, since they are not essential for understanding the content of papers.

**"Content of this section follows Chapter 13 of (Dickey [2003])." Same as before: why this material should be presented in the Summary with full details, borrowed from somewhere else. What is the role of Lemmas 3 and 4? Why are they called Lemmas?**

The second thing I wanted to point out is equivalence of mKP, half of 2d Toda lattice hierarchy and discrete version of KP in which there is a shift operator instead of pseudodifferential one.

**In general, my suggestion would be to rewrite the Summary (and the Introduction in the Thesis) by substantially shortening contents of Chapters 2-4 and to extend Chapter 1 to a proper review of the relevant previous results with all the necessary references.**

Since at this point I can't change anything in my HSE version of a thesis and I don't want them to differ significantly in structure. Moreover at the point of writing the first Chapter I realized that I am not competent enough to write a  complete historical survey of this topic.