Skoltech
Skolkovo Institute of Science and Technology

Jury Member Report – Doctor of Philosophy thesis.

Name of Candidate: Vadim Prokofev

PhD Program: Mathematics and Mechanics

Title of Thesis: Integrable hierarchies of nonlinear differential equations and many-body systems

Supervisor: Professor Anton Zabrodin

Name of the Reviewer: Professor Igor Krichever

I confirm the absence of any conflict of interest (Alternatively, Reviewer can formulate a possible conflict)	**Date: 24-02-2022**

The purpose of this report is to obtain an independent review from the members of PhD defense Jury before the thesis defense. The members of PhD defense Jury are asked to submit signed copy of the report at least 30 days prior the thesis defense. The Reviewers are asked to bring a copy of the completed report to the thesis defense and to discuss the contents of each report with each other before the thesis defense.

If the reviewers have any queries about the thesis which they wish to raise in advance, please contact the Chair of the Jury.

Reviewer's Report

Reviewers report should contain the following items:

- Brief evaluation of the thesis quality and overall structure of the dissertation.
- The relevance of the topic of dissertation work to its actual content
- The relevance of the methods used in the dissertation
- The scientific significance of the results obtained and their compliance with the international level and current state of the art
- The relevance of the obtained results to applications (if applicable)
- The quality of publications

The summary of issues to be addressed before/during the thesis defense

The dissertation by Vadim Prokofev is devoted to an important topic of modern theory of integrable systems which has attracted a considerable interest over the years. Namely, on connections between elliptic solutions of two-dimensional integrable hierarchies and integrable finite dimensional system. A constrained correspondence between rational solutions of the KDV hierarchy and rational Calogero-Moser system was discovered by Airough, Moser and McKean. The equivalence of the theories of elliptic solutions of the KP equations and the elliptic CM system was established in my early work. Later in my joint works with Zabrodin, Wiegman, Babelon and others that result was extended to many other systems.

It is necessary to emphasize that in all these works the isomorphisms of the theories was proved for the first flows of the corresponding hierarchies, only. The extensions of the isomorphism to the level of hierarchies for years had been a well-known open problem which was partially solved only in rational and trigonometric cases by such distinguished researches as T. Shiota.

The solution of this problem obtained by Prokofev is an important contribution to the theory of integrable systems.

The dissertation presents results obtained in a series of 5 papers written by Vadim and Anton Zabrodin. It is aknowledged by the co-author that Vadim's contribution to these works was decisive.

In the Introduction Vadim put the results in a historical context, briefly describes the necessary facts and tools from the theories of two most fundamental hierarchies of the soliton theory: the KP hierarchy and 2D Toda hierarchy, and presents the main results whose detailed proofs are the content of 5 Appendices.

In Appendix A the equivalence of theories of trigonometric solutions of the 2D Toda lattice hierarchy and trigonometric Ruijsenaars–Schneider hierarchy is established.

Appendix B is devoted to the proof of the equivalence of the theories of trigonometric solutions to the Matrix Kadomtsev-Petviashvili hierarchy and the spin generalization of the trigonometric Calogero-Moser hierarchy.

The result presented in Appendix C is outstanding. It establishes the equivalence of the theories of the elliptic solutions of the KP hierarchy and the theory of the elliptic Calogero-Moser system.

In Appendix D that result was extended to the case of elliptic solutions of the 2D Toda hierarchy and the elliptic RS system.

Appendix E the equivalence of hierarchies of the elliptic solutions of the matrix KP hierarchy and the hierarchy of spin CM system is established.

The Doctoral Thesis by Vadim Prokofev contains important result in actively developing area. It satisfies all the necessary requirements and Vadim is worthy of the PhD degree.

Igor Krichever,

Director Center of Advanced Studies, Professor Skolkovo Institute for Science and Technology,

Professor Columbia University,New York

Provisional Recommendation

X ***I recommend that the candidate should defend the thesis by means of a formal thesis defense***

☐ *I recommend that the candidate should defend the thesis by means of a formal thesis defense only after appropriate changes would be introduced in candidate's thesis according to the recommendations of the present report*

☐ *The thesis is not acceptable and I recommend that the candidate be exempt from the formal thesis defense*