Skoltech
Skolkovo Institute of Science and Technology

Jury Member Report – Doctor of Philosophy thesis.

Name of Candidate: Vladimir Fanaskov

PhD Program: Mathematics and Mechanics

Title of Thesis: Statistical inference and machine learning in numerical linear algebra

Supervisor: Associate Professor Aslan Kasimov

Name of the Reviewer: Alexey Zaytsev

I confirm the absence of any conflict of interest ~~(Alternatively, Reviewer can formulate a possible conflict)~~	**Date:** 05.01.2022

The purpose of this report is to obtain an independent review from the members of PhD defense Jury before the thesis defense. The members of PhD defense Jury are asked to submit signed copy of the report at least 30 days prior the thesis defense. The Reviewers are asked to bring a copy of the completed report to the thesis defense and to discuss the contents of each report with each other before the thesis defense.

If the reviewers have any queries about the thesis which they wish to raise in advance, please contact the Chair of the Jury.

Reviewer's Report

Reviewers report should contain the following items:

- Brief evaluation of the thesis quality and overall structure of the dissertation.
- The relevance of the topic of dissertation work to its actual content
- The relevance of the methods used in the dissertation
- The scientific significance of the results obtained and their compliance with the international level and current state of the art
- The relevance of the obtained results to applications (if applicable)
- The quality of publications

The summary of issues to be addressed before/during the thesis defense

- Brief evaluation of the thesis quality and overall structure of the dissertation.

  In the present work, Vladimir studies the utility of techniques from Bayesian statistics, probability theory, and machine learning for the construction of novel solvers for large sparse linear systems. The manuscript is neat and well organized. The first two parts develop Bayesian and machine learning techniques respectively; the appendix supplies additional information on linear systems used for benchmarks and classical relaxation techniques; there is a separate section with proofs of all the propositions and theorems from the main text.
  The first part on Bayesian statistics explains:
  1. How to use graphical models (Gauss-Markov random field) to construct relaxation techniques for the multigrid method
  2. How to calibrate probabilistic projection methods
  3. The way symmetry transformations with indifference principle can be combined to supply probabilistic uncertainty to deterministic numerical algorithms.

  The second part on Machine learning offers:
  1. The construction of optimal BPX preconditioners by loss minimization.
  2. Operator-free multigrid architecture.
  3. Online optimization of relaxation methods with reinforcement learning.

- The relevance of the topic of dissertation work to its actual content

  The actual content of the dissertation precisely corresponds to the topic, while the title can be more precise.

- The relevance of the methods used in the dissertation

  The topic of the dissertation is interdisciplinary since it lies at the intersection of Bayesian statistics, machine learning, scientific computing, and numerical linear algebra. As such, Vladimir used a variety of theoretical and applied techniques including hierarchical Bayesian modeling, stochastic estimation of spectral radius, multivariate normal and matrix normal models, automatic differentiation, variational inference, multilevel methods, PDE discretizations, e.t.c. All used methods are proved to be relevant by the evidence presented in the thesis.

- The scientific significance of the results obtained and their compliance with the international level and current state of the art

  Vladimir contributed several original results:
  1. Gaussian belief propagation solver for non-symmetrical matrices, including block version and acceleration with the geometric multigrid method.
  2. Novel probabilistic reconstruction with improved uncertainty quantification.
  3. Original scheme to introduce probabilistic uncertainty quantification for deterministic algorithms.
  4. Automatic construction of multilevel preconditioners.
  5. Improved operator-free multigrid architecture suitable for machine learning applications and ML-oriented techniques.
  6. Online optimization of iterative methods.

Introduced solvers were compared with state-of-the-art methods where appropriate. For example, the Gaussian belief propagation solver was compared with geometric multigrid schemes with a variety of classical relaxation techniques including Chebyshev, block Gauss-Seidel, and ILU, as well as preconditioned projection techniques.

- The relevance of the obtained results to applications (if applicable)

  As was shown by the author, the results presented in the thesis can be used to construct solvers for linear systems originating from local PDE discretizations (e.g., finite-element, finite-difference-finite volume discretizations). Besides the applications to problems of continuum mechanics, Vladimir also demonstrated how to apply uncertainty quantification for linear solvers in the context of PDE-constrained optimization.

- The quality of publications

  Results from most chapters of the thesis are published in well-known journals including Statistics and Computing, SIAM Journal on Scientific computing, and Journal of Computational and Applied Mathematics.

The summary of issues to be addressed before/during the thesis defense

The thesis has some misprints, but they don't affect the overall positive impression about the work. For example,

- P. 1: the title has 2021 year instead of 2022
- P. 145: proposed algorithms we on a more serious examples -> proposed algorithms on more serious examples
- P. 188: prove it's form -> prove its form

Provisional Recommendation
I recommend that the candidate should defend the thesis by means of a formal thesis defense
I recommend that the candidate should defend the thesis by means of a formal thesis defense only after appropriate changes would be introduced in candidate's thesis according to the recommendations of the present report
The thesis is not acceptable and I recommend that the candidate be exempt from the formal thesis defense